

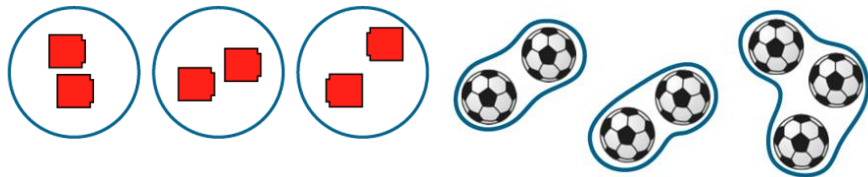
Multiplication and Division

Year 2

Multiplication as Repeated Addition

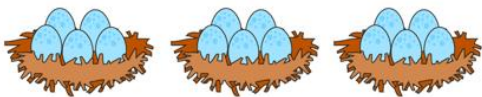
Vocabulary:

Group Equal Unequal Repeated Addition Multiplication Expression Equation
Part Altogether Represents Amount Size



Understand the difference between equal and unequal groups.

The ___ have been grouped.



We can represent equal groups as repeated addition.

There are 3 groups of 5.

$$5 + 5 + 5$$

$$3 \times 5$$

$$5 + 5 + 5 = 3 \times 5$$

We can represent repeated addition using a multiplication expression.

The 3 represents the number of groups.

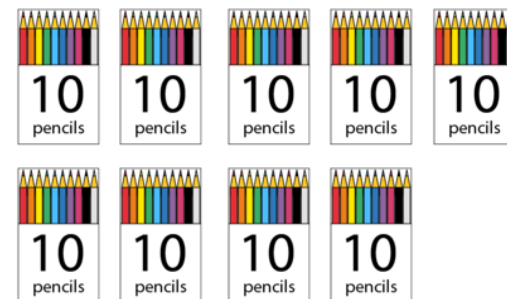
The 5 represents the number of eggs in each group.

15 represents the total number of eggs.

The ___ represents the number of groups.

The ___ represents the number of ___ in each group.

___ represents the total number of ___.



$$9 \times 10$$

We can skip count in multiples of ___ to work out the total amount.

10, 20, 30, 40 ... there are 90 pencils altogether.



$$7 \times 2$$

Notice how the representations allow the children to see each of the numbers (i.e. 10 pencils and 9 packets).

Multiplication and Division

Year 2

Grouping problems: missing factors and division

Vocabulary:

Multiplication Division Factor 'divided by' Represents Skip Counting

Multiplication facts Groups Amount Size



$$\boxed{3} \times 5 = 15$$

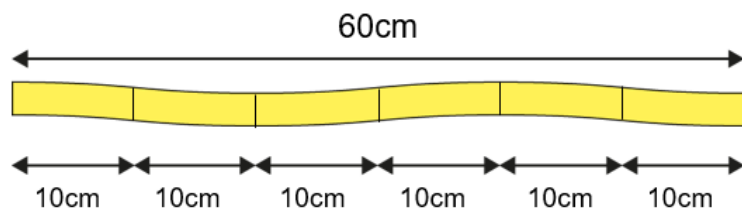
$$15 \div 5 = \boxed{3}$$

We can solve division problems by finding missing factors.

The 15 represents the number of biscuits.

The 5 represents the number of biscuits in each bag (group).

The 3 represents the number of bag (groups).



The 60cm represents the length of the ribbon.
The 10 represents the size of each piece.
The 6 represents the number of pieces we can make.

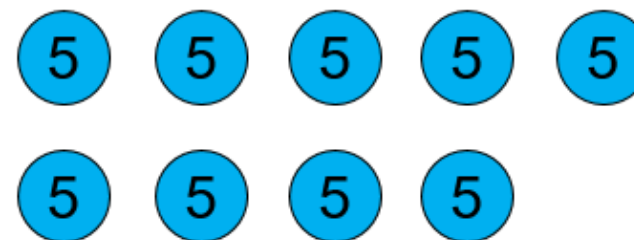
We can use \div to mean 'divided by'

We can use our knowledge of times tables to help solve division problems.

$$\boxed{6} \times 10 = 60$$

$$60 \div 10 = \boxed{6}$$

$$45 \div 5 = \boxed{9}$$



Multiplication and Division

Year 3

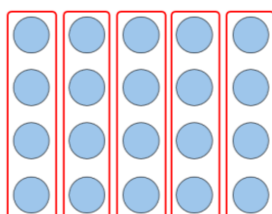
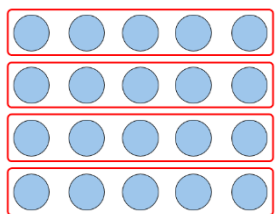
Multiplication and Division Structures

Vocabulary:

Multiplication Division Commutative Grouping (Quotative) Sharing (Partitive)

'Divided into' 'Divided between' 'Divided by' Equation Factor Product

30	÷	5	=	6
dividend	÷	divisor	=	quotient

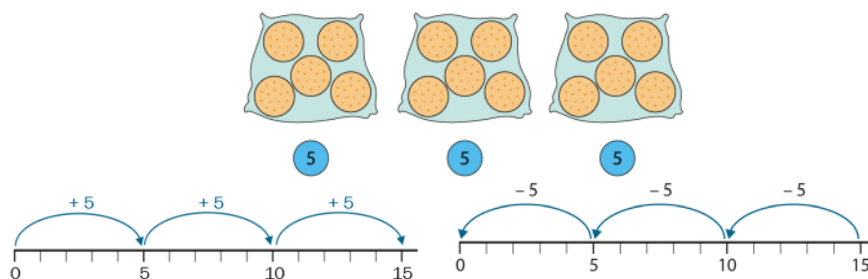


Identify that multiplication is commutative.

$$4 \times 5 = 5 \times 4$$

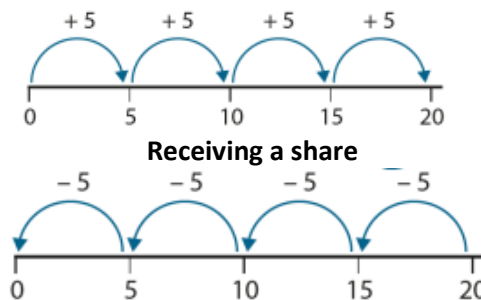
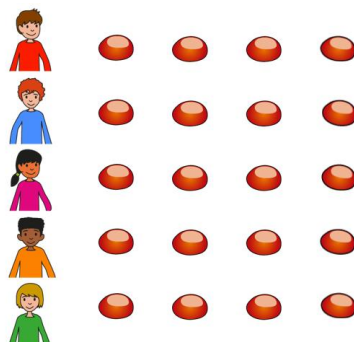
Factor times factor is equal to product.

The order of the factors does not affect the product.



Making groups of

Removing groups of



Receiving a share

Giving a share

Division equations can be used to represent 'grouping' problems.

We can use multiplication facts to find the number of groups.

(Quotative division)

15 divided into groups of 5 is equal to 3.

$$5 + 5 + 5 = 15$$

$$15 - 5 - 5 - 5 = 0$$

$$15 \div 5 = 3$$

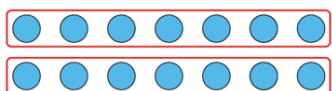
Division equations can be used to represent 'sharing' problems.

We can use multiplication facts to find the size of groups.

(Partitive division)

Four fives are four each.
20 divided between 5 is equal to 4 each.

$$20 \div 5 = 4$$

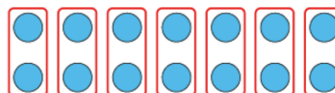


$$14 \div 2 = 7$$

14	
7	7

The same equation can be represented in both grouping and sharing contexts.

7 times 2 is 14, so $14 \div 2 = 7$



$$14 \div 2 = 7$$

14						
2	2	2	2	2	2	2

Multiplication and Division

Year 4

Multiplying and Dividing by 10 and 100

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size
 One-hundredth the size Gattegno chart Factor Product Multiple
 Groups of Inverse

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Develop language in order to multiply and divide by 10 or 100.

80 is ten times bigger than 8.

8 is ten times smaller than 80.

80 is ten times the size of 8

8 is one-tenth the size of 80.

800 is one hundred times bigger than 8.

8 is one hundred times smaller than 800.

800 is one hundred times the size of 8

8 is one-hundredth the size of 80.

$$8 \times 1 = 8$$

$$8 \times 1 \text{ ten} = 8 \text{ tens}$$

$$8 \times 1 \text{ hundred} = 8 \text{ hundreds}$$

Generalisations

All multiples of 10 have a ones digit of zero.

All multiples of 100 have both a tens and ones digit of zero.

To find the inverse of ___ times as many, you divide by ___.

If one factor is made ___ times bigger/smaller then the product will be ten times bigger/smaller

8 made ___ times the size is ___.

ten times the size
x10

1,000s	100s	10s	1s
			8
		8	0

÷10

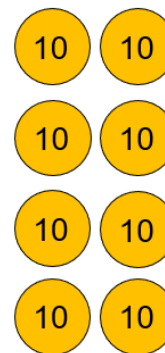
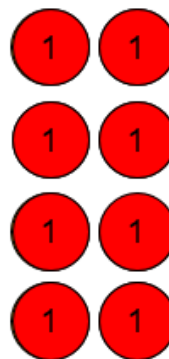
one-tenth of the size

one hundred times the size
x100

1,000s	100s	10s	1s
			8
	8	0	0

÷100

one-hundredth of the size



$$8 \times 1 = 8 \quad 8 \times 10 = 80 \quad 8 \times 100 = 800$$

8 groups of ___ is ___.

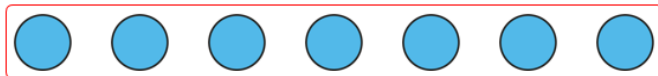
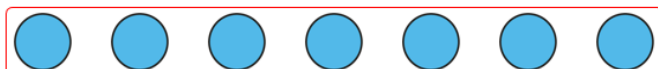
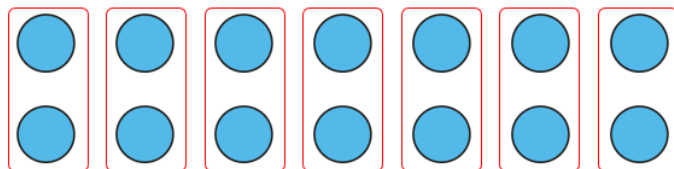
Multiplication and Division

Year 4

Manipulating the Multiplicative Relationship

Vocabulary:

Multiply Divide Commutative Groups of Times Equal to Factors
Product Quotient Dividend Divisor Represents Array



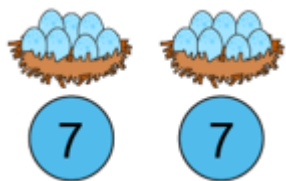
$$2 \times 7 = 7 \times 2$$

Understand that multiplication is commutative and the factors can be

2 groups of 7 is equal to 14.

2, 7 times is equal to 14.

2 groups of 7 is equal to 7, two times.



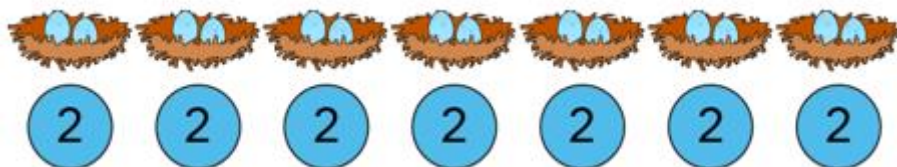
$$2 \times 7 = 14$$

$$7 \times 2 = 14$$

The 2 represents ____.

The 7 represents ____.

The 14 represents ____.



Match equations to representations and contexts.

$$2 \times 7 = 14$$

2 groups of 7



$$7 \times 2 = 14$$

7 groups of 2



$$14 \div 7 = 2$$

2 groups of 7



$$14 \div 2 = 7$$

7 groups of 2



Multiplication and Division

Year 4

The Distributive Property of Multiplication

Vocabulary:

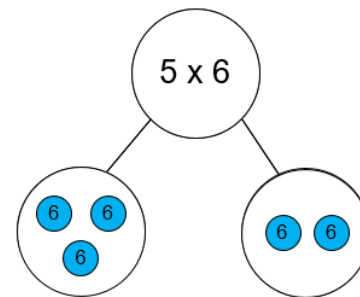
Multiplication Distributive Law Adjacent Multiples Factors Partitioning
Equations Expressions Arrays Part-whole model Difference

$0 \times 6 = 0$	$6 \times 0 = 0$
$1 \times 6 = 6$	$6 \times 1 = 6$
$2 \times 6 = 12$	$6 \times 2 = 12$
$3 \times 6 = 18$	$6 \times 3 = 18$
$4 \times 6 = 24$	$6 \times 4 = 24$
$5 \times 6 = 30$	$6 \times 5 = 30$
$6 \times 6 = 36$	$6 \times 6 = 36$
$7 \times 6 = 42$	$6 \times 7 = 42$
$8 \times 6 = 48$	$6 \times 8 = 48$
$9 \times 6 = 54$	$6 \times 9 = 54$
$10 \times 6 = 60$	$6 \times 10 = 60$
$11 \times 6 = 66$	$6 \times 11 = 66$
$12 \times 6 = 72$	$6 \times 12 = 72$

\times	1	2	3	4	5	6
1	●	●	●	●	●	●
2	●	●	●	●	●	●
3	●	●	●	●	●	●
4	●	●	●	●	●	●
5	●	●	●	●	●	●

$$4 \times 6 + 6$$

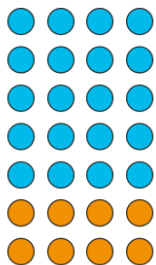
Five sixes is one more six than four sixes.



$$3 \times 6 + 2 \times 6 = 5 \times 6$$

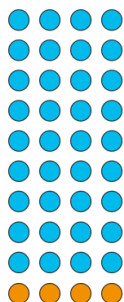
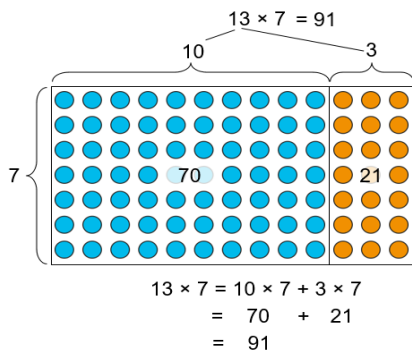
5 is equal to 3 plus 2, so 5 sixes is equal to 3 sixes plus 2 sixes.

Adjacent multiples of ____ have a difference of ____.



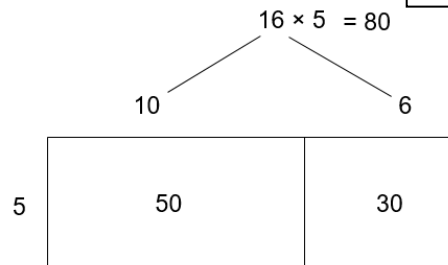
$$\begin{aligned} 7 &= 5 + 2 \\ 7 \times 4 &= 5 \times 4 + 2 \times 4 \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

We can partition one of the factors to make calculations easier.

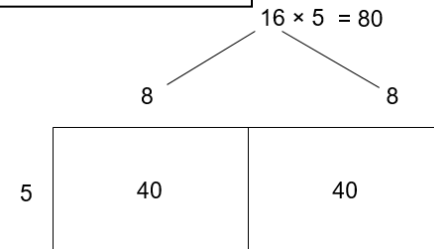


$$\begin{aligned} 9 &= 10 - 1 \\ 9 \times 4 &= 10 \times 4 - 1 \times 4 \\ &= 40 - 4 \\ &= 36 \end{aligned}$$

We can partition the factors in different ways to make calculations easier.



$$\begin{aligned} 16 \times 5 &= 10 \times 5 + 6 \times 5 \\ &= 50 + 30 \\ &= 80 \end{aligned}$$



$$\begin{aligned} 16 \times 5 &= 8 \times 5 + 8 \times 5 \\ &= 40 + 40 \\ &= 80 \end{aligned}$$

Multiplication and Division

Year 5

Multiplying and Dividing by 10 and 100 (1)

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size
One-hundredth the size Gattegno chart Factor Product Multiple Groups of
Inverse Ones Tens Hundreds Tenths Hundredths

$$8 \div 10 =$$

$$0.8 \div 10 =$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$\div 10$
 $\div 10$
 one-tenth
 the size

$$0.08 \times 10 =$$

$$0.8 \times 10 =$$

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$\times 10$
 $\times 10$
 ten times
 the size

We can multiply and divide a number by 10.

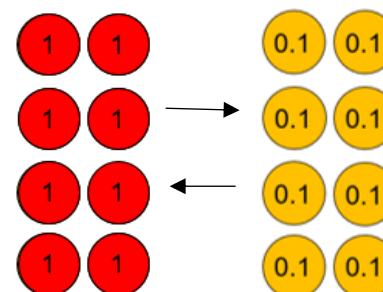
8, made one-tenth the size is 0.8.

8 divided by 10 is 0.8.

First we had 8 ones, now we have 8 tenths.

$$8 \div 10 = 0.8$$

one-tenth of the size



$$0.8 \times 10 = 8$$

ten times the size

We can multiply and divide a number by 100.
Multiplying by 100 is the same as
multiplying/dividing by 10 twice.

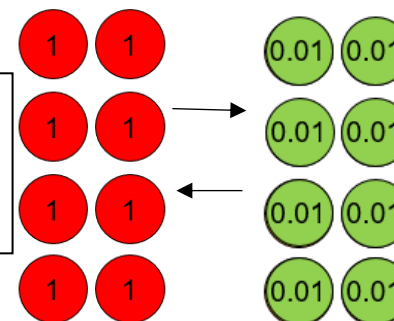
8, made 100 times smaller is 0.08.

8 divided by 100 is 0.08.

First we had 8 ones, now we have 8 hundredths

$$8 \div 100 = 0.08$$

one-hundredth of the size



$$0.08 \times 100 = 8$$

one hundred times the size

Multiplication and Division

Year 5

Multiplying and Dividing by 10 and 100 (2)

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size
One-hundredth the size Gattegno chart Factor Product Multiple Groups of
Inverse Ones Tens Hundreds Tenths Hundredths

$$3.6 \times 10 = 36$$

$$36 \div 10 = 3.6$$

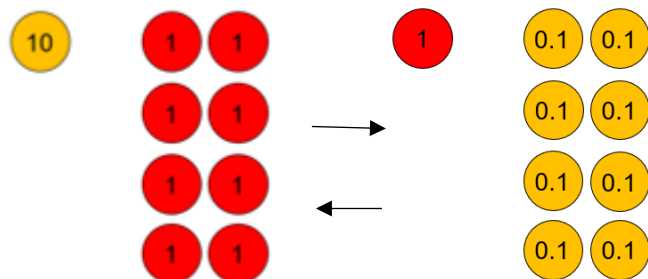
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

$$18 \div 10 = 1.8$$

one-tenth of the size

1.8 is one-tenth the size of 18

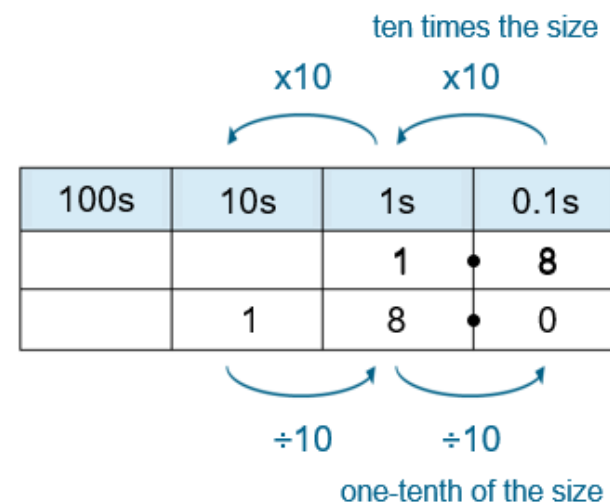
18 divided by 10 is 1.8.



$$1.8 \times 10 = 18$$

ten times the size

__ divided by 10/100 is equal to __.
__ is one-tenth/hundredth the size of __.
__ multiplied by 10/100 is equal to __.
__ is 10/100 times the size of __.



We can multiply and divide numbers with digits greater than 0 by 10 or 100.

Generalisation

To multiply by 10, move each digit one place to the left.

To multiply by 100, move each digit two places to the left.

To divide by 10, move each digit one place to the right.

Multiplication and Division

Year 5

Multiplying and Dividing by 10 and 100 (3).

Vocabulary:

Multiply Divide Unitise Ten/Hundred times Bigger Smaller One-tenth the size
One-hundredth the size Gattegno chart Factor Product Multiple Groups of
Inverse Ones Tens Hundreds Tenths Hundredths

$$0.27 \times 10 = 2.7$$

$$2.7 \div 10 = 0.27$$

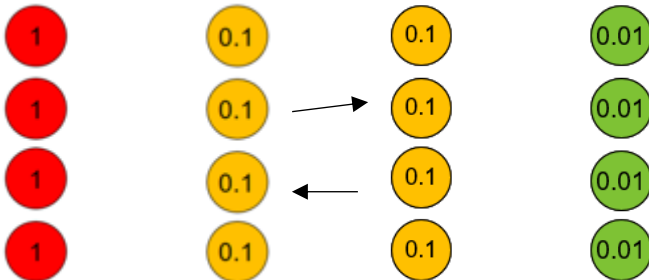
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

0.27 is one-tenth the size of 2.7

2.7 divided by 10 is 0.27.

$$4.4 \div 10 = 0.44$$

one-tenth of the size



$$0.44 \times 10 = 4.4$$

ten times the size

__ divided by 10/100 is equal to __.
__ is one-tenth/hundredth the size of __.
__ multiplied by 10/100 is equal to __.
__ is 10/100 times the size of __.

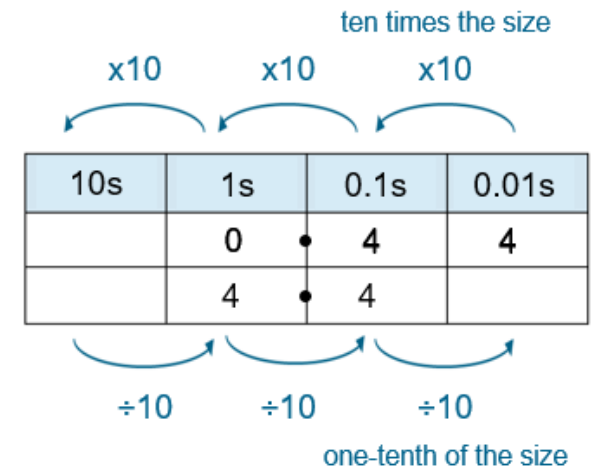
We can multiply and divide numbers with digits greater than 0 by 10 or 100.

Generalisation

To multiply by 10, move each digit one place to the left.

To multiply by 100, move each digit two places to the left.

To divide by 10, move each digit one place to the right.



Multiplication and Division

Year 5

Find Factors and Multiples

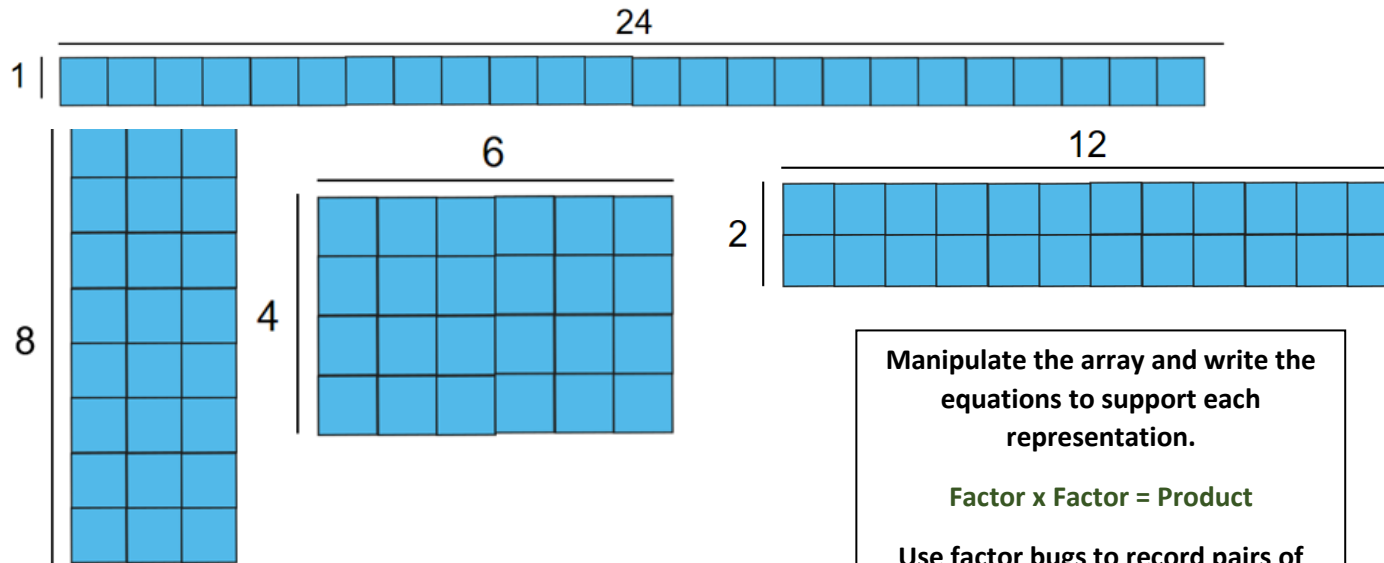
Vocabulary:

Factor Multiple Composite Square Prime Common Factor Prime Factor

Factor Bug Array Positive Integer Working Systematically

Factor \times Factor = Product

Dividend \div Divisor = Quotient



$$8 \times 3 = 24$$

$$4 \times 6 = 24$$

$$2 \times 12 = 24$$

$$1 \times 24 = 24$$

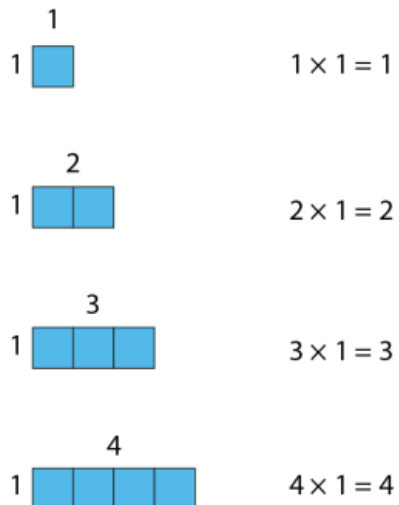
There are ___ tiles. There are ___ rows and ___ columns. So ___ and ___ are factors of ___.

Generalise: Numbers that have more than two factors are composite numbers.

Manipulate the array and write the equations to support each representation.

Factor \times Factor = Product

Use factor bugs to record pairs of factors.



Generalise:

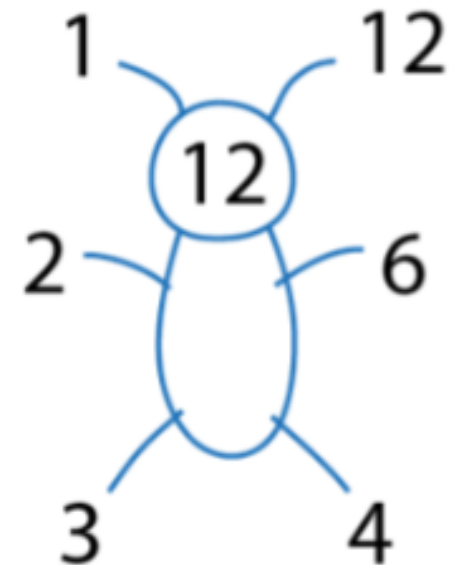
When one is a factor, the product is equal to the other factor.

All positive integers have a factor of 1.

Every positive integer is a factor of itself.

The smallest factor of a positive integer is always 1.

The largest factor of a positive integer is always itself.



Multiplication and Division

Year 5

Find Factors and Multiples

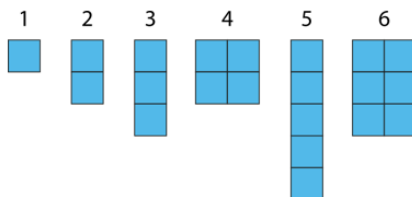
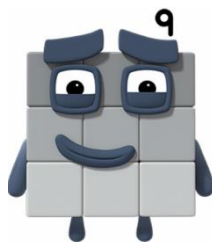
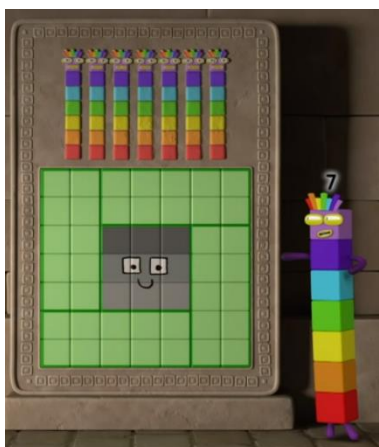
Vocabulary:

Factor Multiple Composite Square Prime Common Factor Prime Factor

Factor Bug Array Positive Integer Working Systematically

Factor x Factor = Product

Dividend ÷ Divisor = Quotient



Extend this to square numbers, and prime numbers recognising the number of factors.

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Make connections with factors and times tables. Make connections with factors of factors

___ is a factor of ___ because it is in the ___ times table.

Nine is a factor of all of these numbers.

Three is a factor of nine which means it is also a factor of all of these numbers.

Is 9 a factor of 54?

$$54 \div 9 = 6$$

9 and 6 are factors of 54.

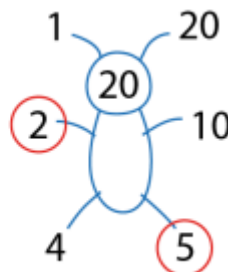
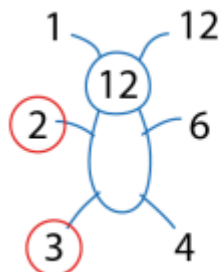
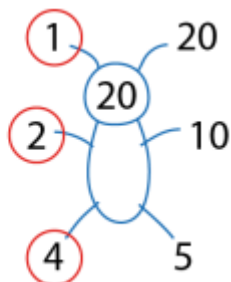
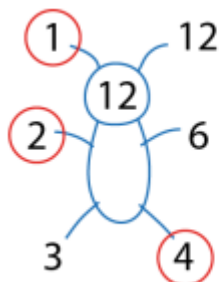
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Use factor bugs to find

common factors

and

prime factors.



Multiplication and Division

Year 5

Find Factors and Multiples

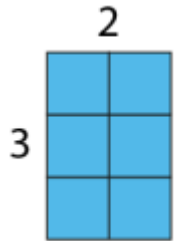
Vocabulary:

Factor Multiple Composite Square Prime Common Factor Prime Factor

Factor Bug Array Positive Integer Working Systematically

Factor x Factor = Product

Dividend ÷ Divisor = Quotient



Introduce Multiples

___ is a factor of ___ because ___ x ___ = ___.

___ is a multiple of ___ because ___ x ___ = ___

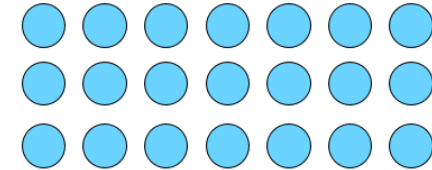
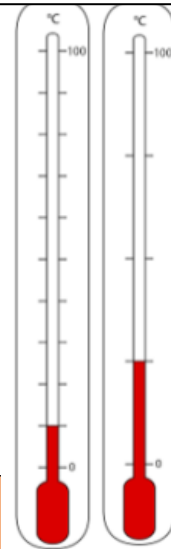
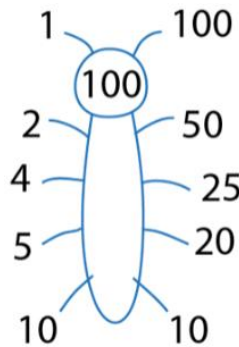
___ is a factor of ___ because ___ ÷ ___ = ___.

___ is a multiple of ___ because ___ ÷ ___ = ___

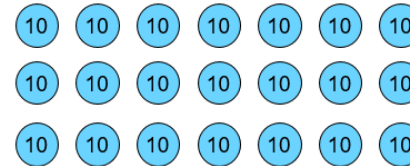
Identify Common Multiples using a 100 square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Factors of 100 can be applied to contexts



$$7 \times 3 = 21$$



$$7 \times 30 = 210$$

$$70 \times 3 = 210$$

$$10 \times 21 = 210$$

Make statements about factors and multiples whilst increasing the amount of each counter in the array.

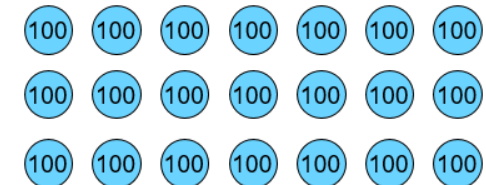
___ represents the number of counters in each row.

___ represents the total value of the counters in each column.

___ represents the total value of the counters.

3, 7, 10, 21 and 70 are factors of 210.

210 is a multiple of 3, 7, 10, 21 and 70.



$$7 \times 300 = 2,100$$

$$700 \times 3 = 2,100$$

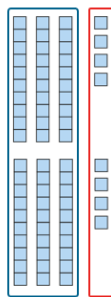
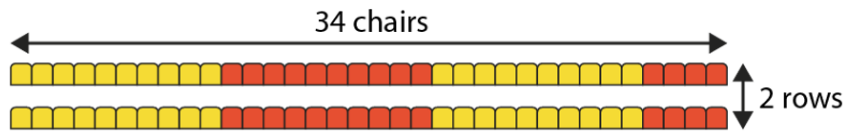
$$100 \times 21 = 2,100$$

Multiplication and Division

Year 5

Vocabulary:

Ones Tens Hundreds Thousands Represents Partition Recombine

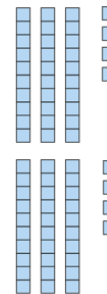


$$34 = 30 + 4$$

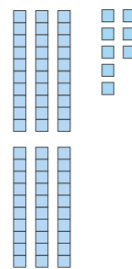
$$34 \times 2 = 30 \times 2 + 4 \times 2$$

Use dienes to represent context as repeated addition and move to multiplication.

Move between representations of dienes and expanded written multiplication.

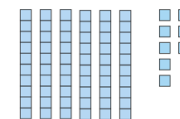


10s	1s
3	4
×	
	2



10s	1s
3	4
×	
	2
	8

$$2 \times 4 \text{ ones} = 8 \text{ ones}$$



10s	1s
3	4
×	
	2
	8
6	0

$$2 \times 4 \text{ ones} = 8 \text{ ones}$$

$$2 \times 3 \text{ tens} = 6 \text{ tens}$$

321 × 3 = 963

100s	10s	1s
3	2	1
×		
		3
	6	0
9	0	0
9	6	3

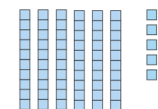
3 2 1

× 3

9 6 3

Represent 3 digit multiplication without exchanges using dienes, moving from expanded layout to compact layout.

Move between representations of expanded layout and compact layout.



10s	1s
3	4
×	
	2
	8
6	0
6	8

$$2 \times 4 \text{ ones} = 8 \text{ ones}$$

$$2 \times 3 \text{ tens} = 6 \text{ tens}$$

Multiplication and Division

Vocabulary:

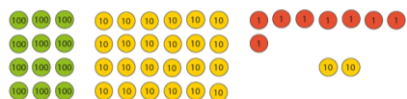
$$367 \times 4 = \boxed{}$$



1,000s	100s	10s	1s
	3	6	7
			4

Represent 3 digit by 1 digit multiplication with exchanges using place value counters, moving from expanded layout to compact layout.

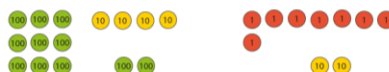
$$367 \times 4 = \boxed{}$$



$4 \times 7 \text{ ones} = 28 \text{ ones}$
 $= 2 \text{ tens} + 8 \text{ ones}$

1,000s	100s	10s	1s
	3	6	7
			4
		2	8

$$367 \times 4 = \boxed{}$$



$4 \times 7 \text{ ones} = 28 \text{ ones}$
 $= 2 \text{ tens} + 8 \text{ ones}$
 $4 \times 6 \text{ tens} = 24 \text{ tens}$
 $= 2 \text{ hundreds} + 4 \text{ tens}$

1,000s	100s	10s	1s
	3	6	7
			4
		2	8
	2	4	0

$$367 \times 4 = \boxed{}$$



$4 \times 7 \text{ ones} = 28 \text{ ones}$
 $= 2 \text{ tens} + 8 \text{ ones}$
 $4 \times 6 \text{ tens} = 24 \text{ tens}$
 $= 2 \text{ hundreds} + 4 \text{ tens}$
 $4 \times 3 \text{ hundreds} = 12 \text{ hundreds}$
 $= 1 \text{ thousand} + 2 \text{ hundreds}$

1,000s	100s	10s	1s
	3	6	7
			4
		2	8
	2	4	0
1	2	0	0

$$367 \times 4 = \boxed{1,468}$$

Move from expanded layout to compact layout.

$$\begin{array}{r} 367 \\ \times 4 \\ \hline 1468 \\ \hline 22 \end{array}$$

- If there are 10 or more ones, we must regroup ones into tens and ones.
- If there are 10 or more tens, we must regroup into hundreds and tens.
- If there are 10 or more hundreds, we must regroup into thousands and hundreds.

Multiplication and Division

Vocabulary:

Partitive (sharing) Quotitive (grouping) Ones Tens Hundreds Thousands

84	÷	4	=	21	$\begin{array}{r} 21 \\ 4 \overline{) 84} \end{array}$
dividend	÷	divisor	=	quotient	$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$

Use sticks to represent partitive (sharing) context where the dividend is divisible (to give a whole number). Skip count in multiples of the divisor.

84 sticks shared equally between 4 children. How many sticks each?

$$84 \div 4 = \square$$

Step 1 – write the divisor and dividend:



10s 1s

$$\begin{array}{r} 4 \overline{) 84} \end{array}$$

Step 2 – share the 10s:



10s 1s

$$\begin{array}{r} 2 \\ 4 \overline{) 84} \end{array}$$

8 tens ÷ 4 = 2 tens

8 tens divided by 4 is equal to 2 tens.

Remove a group of 4 each time before sharing.

One four is one each. That's four.

Two fours is two each. That's eight.

Eight divided between 4 is equal to two each.

Each child gets two sticks.

Step 3 – share the 1s:



10s 1s

$$\begin{array}{r} 21 \\ 4 \overline{) 84} \end{array}$$

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

Add the partial quotients to find the quotient.

2 tens + 1 one = 21

10s 1s

$$\begin{array}{r} 21 \\ 4 \overline{) 84} \end{array}$$

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

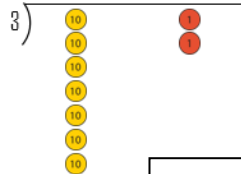
84	÷	4	=	21	$\begin{array}{r} 21 \\ 4 \overline{) 84} \end{array}$
dividend	÷	divisor	=	quotient	$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$

72 sticks shared equally between 3 children. How many sticks each?

$$72 \div 3 = \square$$

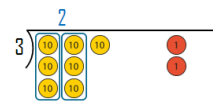
Step 1 – write the divisor and the dividend:

$$3 \overline{) 72}$$



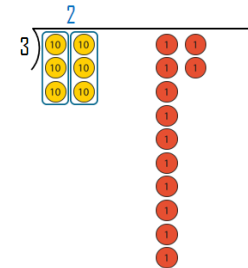
Step 2 – share the 10s:

$$3 \overline{) 72} \quad \begin{array}{r} 2 \\ 3 \overline{) 72} \end{array}$$



Step 3 – exchange:

$$3 \overline{) 72} \quad \begin{array}{r} 2 \\ 3 \overline{) 72} \end{array}$$



$$7 \text{ tens} \div 3 = 2 \text{ tens r } 1 \text{ ten}$$

$$7 \text{ tens} \div 3 = 2 \text{ tens r } 1 \text{ ten}$$

Use sticks and place value counters to represent partitive (sharing) context where the dividend is divisible (to give a whole number) though requires an exchange from the tens. Skip count in multiples of the divisor.

If dividing the tens gives a remainder of one or more ten, we must exchange the remaining tens for ones.

$$72 \div 3 = \boxed{24}$$

Step 4 – share the 1s:

$$3 \overline{) 72} \quad \begin{array}{r} 24 \\ 3 \overline{) 72} \end{array}$$

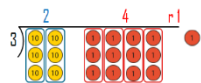


$$7 \text{ tens} \div 3 = 2 \text{ tens r } 1 \text{ ten}$$

$$12 \text{ ones} \div 3 = 4 \text{ ones}$$

$$73 \div 3 = \boxed{24 \text{ r } 1}$$

$$3 \overline{) 73} \quad \begin{array}{r} 24 \text{ r } 1 \\ 3 \overline{) 73} \end{array}$$



$$7 \text{ tens} \div 3 = 2 \text{ tens r } 1 \text{ ten}$$

$$13 \text{ ones} \div 3 = 4 \text{ ones r } 1 \text{ one}$$

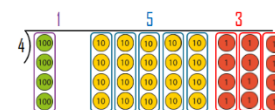
Apply the same representations though this time include a remainder.

Then extend to division of 3 digits by one digit and where there can be no hundreds cannot be shared.

If dividing the hundreds gives a remainder of one or more hundred, we must exchange the remaining hundreds for tens.

$$612 \div 4 = \boxed{153}$$

$$4 \overline{) 612} \quad \begin{array}{r} 153 \\ 4 \overline{) 612} \end{array}$$



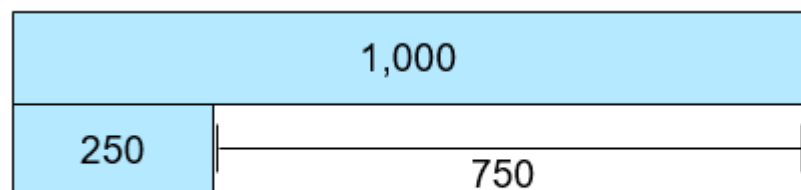
$$6 \text{ hundreds} \div 4 = 1 \text{ hundred r } 2 \text{ hundreds}$$

$$2 \text{ hundreds} = 20 \text{ tens}$$

$$21 \text{ tens} \div 4 = 5 \text{ tens r } 1 \text{ ten}$$

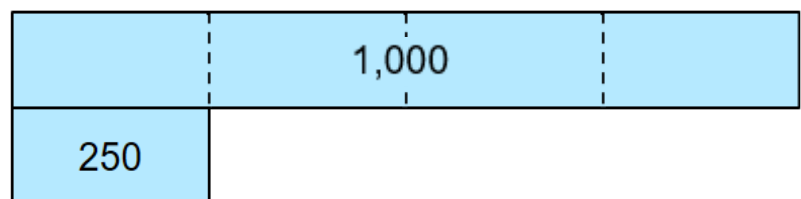
$$1 \text{ ten} = 10 \text{ ones}$$

$$12 \text{ ones} \div 4 = 3 \text{ ones}$$



$$250 + 750 = 1,000$$

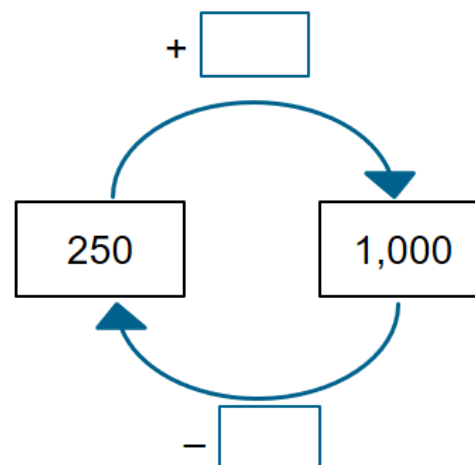
$$1,000 - 750 = 250$$



$$250 \times 4 = 1,000$$

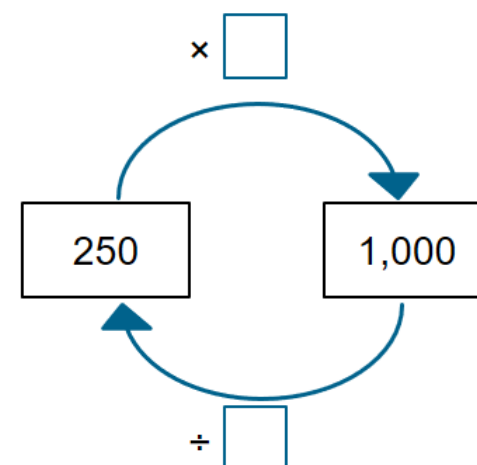
$$1,000 \div 4 = 250$$

The relationship between two numbers can be expressed both additively and multiplicatively.



1000 is ___ more than 250.

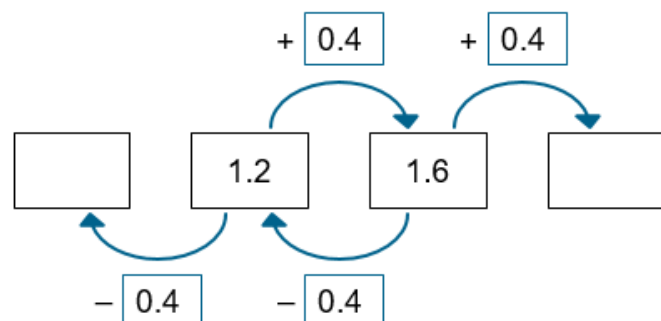
250 is ___ less than 1000.



1000 is ___ times the size of 250.

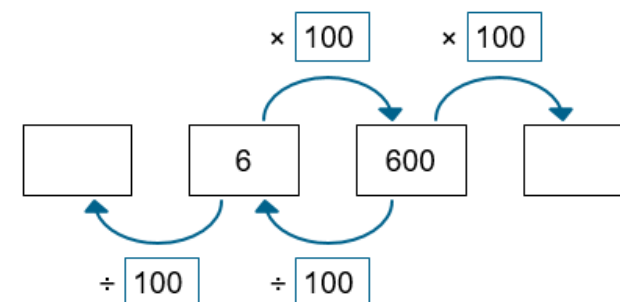
250 is one-_____ of 1000.

To find one-quarter of a number, we divide by 4.

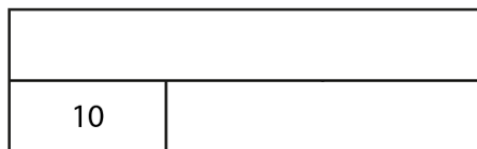


Finding the difference can help calculate the unknown terms in a sequence.

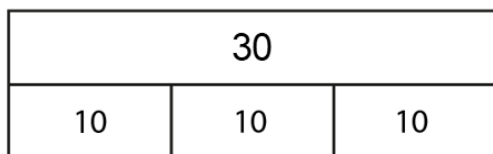
Finding the known multiplier can help calculate the unknown terms in a sequence.



$$\frac{1}{3} \text{ of } ? = 10$$



$$\frac{1}{3} \text{ of } ? = 10$$



$$\frac{1}{3} \text{ of } 30 = 10$$

Calculate the unknown whole by recognising how many parts the whole has been divided into.

$$252 = 3 \times 84$$

$$2,520 = 30 \times \boxed{}$$

$$252 = 3 \times 84$$

$$\boxed{} = 3 \times 85$$

$$252 = 3 \times 84$$

$$252 = 3 \times 60 + 3 \times \boxed{}$$

Manipulate an equation to solve another. Pupils could:

- rearrange the terms;
- rewrite using inverse operations;
- apply place value;
- use the properties of division that correspond to the commutative, associative or distributive property of multiplication;
- use the compensation property.

Additive examples

Multiplicative examples

$$625 - 148 = 477$$

$$6,250 - 1,480 = \boxed{}$$

$$625 - 148 = 477$$

$$625 - 70 - \boxed{} = 477$$

$$625 - 148 = 477$$

$$625 - 248 = \boxed{}$$

$$14.8 + 7.6 = 22.4$$

$$1,480 + \boxed{} = 2,240$$

$$14.8 + 7.6 = 22.4$$

$$\boxed{} - 7.6 = 14.8$$

$$14.8 + 7.6 = 22.4$$

$$12.8 + \boxed{} = 22.4$$

$$4,800 \div 25 = 192$$

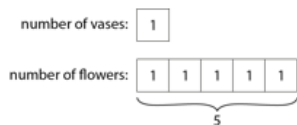
$$25 \times 192 = \boxed{}$$

$$4,800 \div 25 = 192$$

$$4,800 \div 250 = \boxed{}$$

$$4,800 \div 25 = 192$$

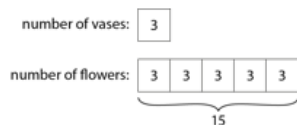
$$4,800 \div 5 \div 5 = \boxed{}$$



$$1 \times 5 = 5$$

$$5 \div 5 = 1$$

$$5 \times \frac{1}{5} = 1$$



$$3 \times 5 = 15$$

$$15 \div 5 = 3$$

$$15 \times \frac{1}{5} = 3$$

Ratio table to compare sets of information.

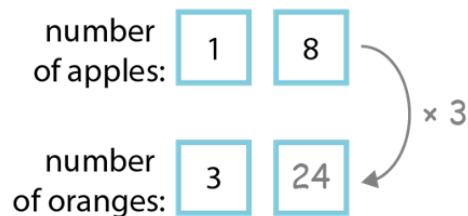
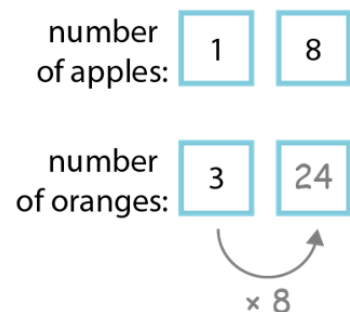
For every __, there are __.

For every 1 litre of petrol, you can drive 7 miles.

For every 7 miles you will drive, you need 1 litre of petrol.

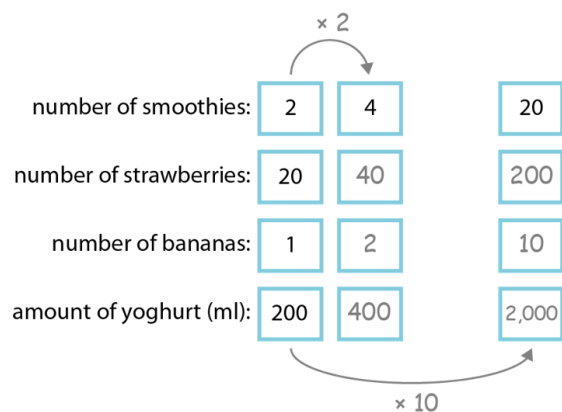
Extend sequences using knowledge of patterns based on ratio table.

Litres of petrol	1	2	3	4	5	6	7	8	9	10
Miles driven	7	14	21	28	35	42	49	56	63	70



Explore vertical and horizontal relationship between numbers.

For every __, there are __.

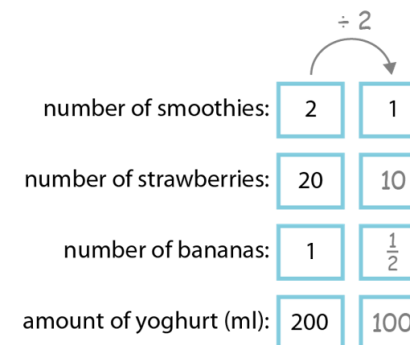


Identify the scale-factor in order to find unknown values.

__ is __ times the size of __.

Therefore I must multiply/divide by __.

__ is one-__ the size of __.



Addition and Subtraction

Year 6

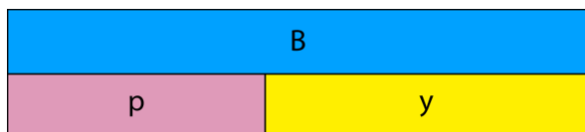
Solve Problems with Two Unknowns

Vocabulary:

Additive Multiplicative Relationship Represents Equation Two Unknowns
Scale-factor Ratio ___ times the size one-___ the size of Total Bar Model
Structure



$$B = \boxed{r} + \boxed{b}$$



$$B = \boxed{p} + \boxed{y}$$

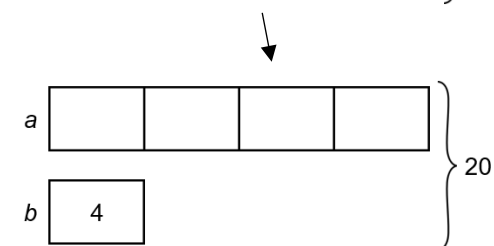
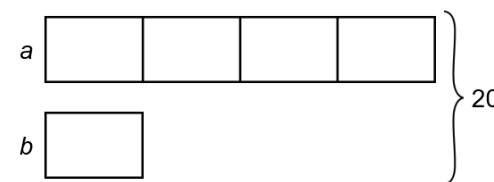
Use Cuisenaire to find 2 bars of total length that are equal to another.

There is more than one solution to the problem.

There can be infinite solutions to a problem.

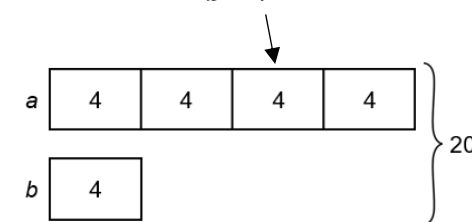
$$5 \times \boxed{} = 10 \times \boxed{}$$

Solve multiplicative problems with two unknowns when the total is known.



$$\text{one part} = 20 \div 5 = 4$$

$$b = 4$$



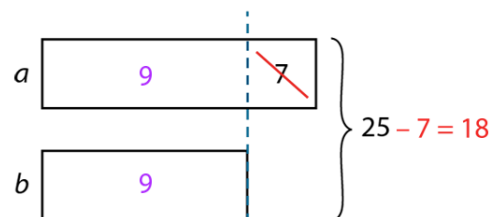
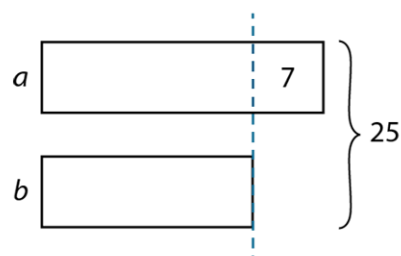
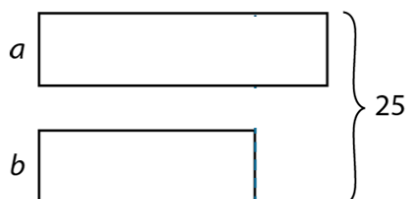
$$\text{one part} = 20 \div 5 = 4$$

$$b = 4$$

$$a = 4 \times 4 = 16$$

The two numbers are 16 and 4.

Solve additive problems with two unknowns when the total is known.



$$b = 18 \div 2 = 9$$

$$a = 9 + 7 = 16$$

The two numbers are 9 and 16.